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Delay Modeling of Un-signalized Roundabouts Using Neural Network and Regression

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Keywords

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Abstract

Delay is one of the primary variables of efficiency of intersections and roundabouts. There are not extensive experiences about delay in roundabouts in comparison with delay at intersections, and there haven't been many studies in Iran particularly about un-signalized roundabouts. The main goal in traffic designs is to have a safe and efficient flow. In order to achieve this goal, a video has been prepared about traffic conditions of three un-signalized roundabouts in Rasht. Data related to geometric designs such as nearside legs width, turning width, far-side legs width and central roundabout diameter were measured by field observation. Data related to traffic volume such as nearside legs volume, turning volume and far-side legs volume were observed by already prepared videos. Eventually, analyses about right turning movements at un-signalized roundabouts were carried out by linear and nonlinear regression models. The best delay model of direct movements has linear relation, and left turning and direct movements has logarithmic linear relation. Because of many variables and the high accuracy of neural network, it was decided to model the data using this method, and the best model is obtained for the prediction of un-signalized roundabouts' delay.

1. Introduction

Increasing growth of population and uncontrolled development of cities have made human and good transfer so complicated. Traffic and transportation system in cities is known as one of the most important infrastructures of countries' economy, actually that is one of the important developmental indexes of countries in transportation. The vast range of issues and challenges of transportation needs planning and considering practical approaches to reduce traffic density and making smooth traffic. The use of new roundabouts as a suitable tool for controlling traffic instead of other kinds of intersections and cause more capacity, safety and less delay in lower and medium volumes [1, 2].

Traffic flow management is surely one of the main issues in a modern society, and government in order to have efficient transportation system have to invest considerably in this field. There are problems such as increasing growth of traffic flow volume, limitation on development and building new infrastructures, environmental issues caused by spreading of pollutions, and unpleasant delays that happen in crowded traffic flow, all of which need to be solved by the planning officials [3]. Traffic delay is an evaluating criterion of controlled intersections efficiency by stop sign and yield sign, signalized intersections and roundabouts [4].

A lot of models have been suggested for calculation and estimation of vehicles delay of signalized and un-signalized intersections for more than 40 years. One of the first models of delay estimation is wardrop model that released in 1952. Wardrop imagined that vehicles arrive at the intersection at the same time. In this model, wardrop found that the term $1/2s$ in comparison with the term r is smaller amount and could be ignored. A few years later, three other models in estimation of caused delay on vehicles were presented by Webster, Miller, and Newell, and later Hutchinson, Sosin, and cronje compared them analytically [5-7]. Hutchinson amended the Webster's delay model by the introduction of variable I. When variable I equals one, the Hutchinson relation will be Webster's. Hutchinson studies showed that the delay achieved from Webster's equation calculates a little amount of delay when is bigger than one and saturation degree is high. He also stated that the improvement of Webster's equation by variable I is a suitable method for random delay estimation based on experimental studies [8-11].

A lot of studies have been done about the delay of signalized intersections in Rasht [12-13]. Although there are

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various un-signalized roundabouts in Rasht, few researches have been conducted about geometric and traffic parameters of roundabouts, particularly about delay in them. In this paper, by observation of right, left and direct movements volumes of vehicles in every nearside legs to a roundabout and volumes of turning movements around a roundabout and consideration of width of nearsides and far-sides of roundabouts, regression models for delay caused to vehicles of three un-signalized roundabouts in Rasht were determined and eventually, these models have been improved by neural networks.

2. Delay Criterion

Traffic delay is one of the evaluating criteria of controlled intersections' function with stop signs, yield signs, signalized intersections and roundabouts. The majority of studies which have been done for development of delay model at controlled intersections are with stop signs and traffic lights, and roundabouts have not been paid much attention. In 1994 HCM, delay model is for controlled intersections including stop signs and traffic lights, but no models are considered for roundabouts. In both HCM 1997 and HCM 2000, there is a method for estimate of roundabout capacity, although there are no models considered for the estimate of delay in roundabout [14 -16].

But delay relation on the vehicles which has been mentioned in HCM 2010 has many nearside variables for calculation of vehicles delay such as cycle length, green time, saturation flow rate, capacity, etc. In this paper, assuming all other variables constant, volumes of nearside vehicles to intersection is just assumed as a variable. Any random variable has a specified frequency distribution with given mean and variance that is one of the characteristics of every distribution frequency [17].

Assuming mean and variance of nearside volume and nearside distribution of vehicle to intersections, the values are clear, it is possible to calculate mean and variance of saturation degree by Eq. (1), on the basis of Eq. (1), the type of frequency distribution of vehicles' saturation rate would be equal with the type of frequency distribution of vehicles' nearside volume to intersection.

$$X = V/C \tag{1}$$

where *X* is the saturation flow rate of near side route to the intersection, *V* the nearside flow rate of the vehicles to the intersection and *C* the capacity of near side route to the intersection.

Given that saturation flow rate, green time of each phase and cycle length are constant, so with the above assumptions the capacity of each nearside route to intersection on the basis of Eq. (2) will be calculated as

$$C = S \times \frac{g_i}{c} \tag{2}$$

In the above relation, *C* stands for the capacity of each nearside route to the intersection, *S* is the saturation flow rate, g_i is the green time related to the intended phase and *c* denotes the cycle length of the traffic light. According to Eq. (1), the average level of saturation rate which is equal to the level of nearside flow rate of the vehicles to the intersection, will be divided by the capacity of nearside route to intersection. Also, on the basis of Eq. (2), the variance of saturation rate which is equal to the variance of nearside flow of the vehicles to intersection is divided by the square of the capacity.

Eq. (3) is HCM 2010 equation of delay function of the vehicles. After the calculation of the expected values of saturation rate, the amount of vehicles' delay on the basis of HCM delay function with the help of Taylor series will be estimated. On the basis of assumptions made in this paper based on Eq. (3), the saturation rate is the only random variable [17].

$$D = 0.5C \times \frac{\left(1-\frac{g}{C}\right)^2}{\left(1-Min(x,1)\frac{g}{C}\right)} + 900T\left[(x-1) + \sqrt{(x-1)^2 + \frac{8kIx}{cT}}\right] + d_3 \tag{3}$$

where C, g, x and c are the cycle length, green time related to the intended phase, saturation degree and capacity, respectively. Delay at the intersections in comparison with roundabout capacity is not paid much attention by the researchers.

3. Methodology

In order to consider effective variables and estimation of delay model at urban un-signalized roundabouts, data of traffic volume, features of geometric design and data of green time at some of un-signalized roundabouts are needed. Data need to be collected in the way that in each period of time (15 minutes) enough travel time observations for various movements at roundabouts should be gathered, and traffic volume in every movement according to types of vehicle is also needed to be gathered. Weather condition and observation time should be the same for all of the roundabouts as much as possible that have been considered for the current study. In this study, three un-signalized roundabouts with different features in Rasht have selected and needed data have been collected about them. These roundabouts were selected among 10 roundabouts in Rasht, all of which are four-leg, and it was tried that firstly the roundabouts in terms of the streets ending to them are different. Secondly, these roundabouts were chosen from different parts of the city. The features of the roundabouts have been explained on the Tables 1 and 2.

There are many methods to count traffic volumes such as field observation, using of mechanical counters and video recording, among which video recording is the most common. In the current study, the method of video recording for counting the volume of vehicles according to the movement and the type of vehicle has been used. This method was used at noon pick hour, then needed information were gathered from the video. In order to study delays at roundabouts and consider effective variables on delays, data of travel times and variety of different movements at roundabouts are needed. In this study, all of the travel times of vehicles during the whole observation time are gathered by the video films. Thus a benchmark for each nearside and far-side of the roundabout is considered and the travel time of intended movements are measured by a chronometer.

Table 1. Turning width and roundabout diameter characteristic of study place

Roundabout	Time of Observation (hour)	Turning Width (meter)	Roundabout Diameter (meter)
1	2	7.30	16.00
2	2	10.95	28.00
3	2	18.25	90.00

Table 2. Cross-section characteristic of study place

Roundabout	Enter Name	Type of Street	Cross-section characteristic						
			Nearside			Median	Far-side		
			Parking Lane	Passing Lane	Total		Parking Lane	Passing Lane	Total
1	A	Collector	2.2	5.8	8.00	3.30	2.2	7.80	10.00
	B	Local	2.2	7.8	10.00	8.50	2.2	6.80	9.00
	C	Collector	2.2	11.3	13.50	6.30	2.2	8.80	11.00
	D	Local	2.2	11.1	13.30	8.00	2.2	14.80	17.00
2	A	Major arterial	2.2	9.05	11.25	0.00	2.2	9.05	11.25
	B	Major arterial	2.2	6.55	8.75	0.00	2.2	6.55	8.75
	C	Major arterial	2.2	8.80	11.00	0.00	2.2	8.80	11.00
	D	Minor arterial	2.2	9.55	11.75	1.00	2.2	9.55	11.75
3	A	Major arterial	2.2	16.40	23.00	0.00	2.2	9.30	9.30
	B	Major arterial	2.2	8.40	10.60	0.00	2.2	8.40	10.60
	C	Major arterial	2.2	6.55	8.75	0.00	2.2	6.55	8.75

4. Statistical Analysis Method

In many cases, researchers conduct study to examine the relationship between two random variables. In these circumstances of the relationship between two quantitative variables, measures called Pearson Correlation Coefficient were used. Usually in investigating the Correlation between two variables, the values are dependent on each other. In analysing the correlation between two variables may not select any of the two variables as the reason for the other. Primarily for the study of harmony between the two variables, researcher looks for measures having the following features

- does not depend on the unit of tow community.
- is bounded.

The data set used in a correlation test including values obtained from the two variables X and Y can be in stated the form of a bivariate random sample $(X_1,Y_1),..., (X_n,Y_n)$. Study of the relationship between variables is performed by correlation analysis, which indicated that there is a linear relationship between two variables. The correlation coefficient formula is as follows

$$r = \frac{Cov\,(X,Y)}{S_x S_y} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{[\sum_{i=1}^{n}(x_i-\bar{x})^2][\sum_{i=1}^{n}(y_i-\bar{y})^2]}} \quad (4)$$

The correlation coefficient is always a value between zero and one ($-1<r<1$) and according to the value of r in different states, will have different interpretations of the relationship between X and Y.

- $r = 1$, in this state, correlation is complete and direct, and increasing the value of x, the value of y definitely increases.
- $r = -1$, in this state, correlation is complete and inverse, and increasing the value of x, the value of y decreases.
- $-1< r < 0$, in this state, correlation is incomplete and inverse, and increasing the value of x, the value of y partially decreases.
- $0 < r < 1$, in this state, correlation is incomplete and direct, and increasing the value of x, the value of y partially increases.
- $r = 0$, there is no linear relationship (For example, the relationship may be a quadratic).

The zero correlation coefficient does not mean there is no relationship between two variables, but there is no linear relationship between two them. The reason for this is that the formula for the Pearson correlation coefficient only measures the linear relationship. In one-way correlation test, the following assumptions are examined (Eq. (5)).

$$1\begin{cases}H_0: r = 0\\ H_1: r > 0\end{cases} \qquad 2\begin{cases}H_0: r = 0\\ H_1: r < 0\end{cases} \quad (5)$$

The direction of inequality is determined according to estimates for sample r. If r is positive, we use test (1) and if it is negative, we use test (2). The value of Pearson correlation coefficient, indicates the severity of the relationship, this measure evaluates the strength of linear relationship between the variables x and y. If the Pearson value is between zero and 0.3, it is weak, between 0.3 and 0.6, it is medium and above 0.6 it is strong. This measure is conventional, but it is usually used in interpreting the results and different studies. It was first introduced by Karl Pearson, an English famous scientist, that's why it is called the Pearson coefficient. This coefficient is defined in the following as

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} \quad (6)$$

In the context of regression, the purpose is to obtain a mathematical relationship between one or more independent variables and a dependent variable in order to predict the dependent variable value using the value(s) of the independent variables. Therefore, in regression, not only is the correlation between several independent variables and a dependent variable is evaluated, but also the type and shape of the mathematical relationship is determined. In simple linear regression, the relationship between the independent variable x and the dependent variable y is in the form of a straight line and the equation is written as $y = \alpha x + \beta$. α and β are called the regression parameters, α is constant and β is the slope of the line. Estimation of α indicates the predicted value of y for $x = 0$ and estimation of β indicates the extent of y change if x increases for one unit. In the linear regression, it is assumed that y is normally distributed.

In the linear regression, dependent variable y_i is a linear compound of coefficients (there is no need to be linear to the independent variables). For example, the following simple regression analysis with N points, dependent variable of x_i and coefficients of β_0 and β_1 is linear according to Eq. (7)

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i \,, i = 1, \dots, N \quad (7)$$

In both cases, ϵ_i is the error value and footnote i shows the number of each observation (each pair of x_i and y_i). Having a set of these points, the model can be obtained by Eq. (8)

$$y_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + e_i \quad (8)$$

Parameter e_i is the residual defined as $e_i = y_i - \hat{y}_i$. The common method to obtain parameters is the method of least squares. In this method, the parameters are obtained by minimizing the following relation

$$SSE = \sum_{i=1}^{N} {e_i}^2 \quad (9)$$

In the case of simple regression, the parameters of this method will be equal to Eqs. (10) and (11) as

$$\hat{\beta}_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} \quad (10)$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \quad (11)$$

Regression is the most widely used statistical methods for the measurement and representation of the relationship model between two variables. Linear regression estimates the coefficients of the linear equation that has the closest match to the observed data. Using the equation of the back line, prediction of subsequent values is also possible. In regression equation, we have an independent variable and a dependent variable and in the multiple regression equation the number of independent variables is more than one. In this study, drawing the scatter plot and curve fitting, the required equations by regression were obtained in SPSS.

5. Data Analysis and Calibration

At first, we need to set the effective variables for each movement at roundabouts and then provide the needed data and obtain the delay estimation model using the linear and nonlinear regression models. Table 3 shows the left, right, direct and turning volume variables in nearside legs, roundabout turning volume and fareside volume and vehicles delay on three studied roundabouts.

5.1. Right Turning Movement Model

At first, we set effective variables on delay time for right turning movement. Effective traffic and geometric variables on right turning delay are V_r, right turning volume in considered nearside, V_c, turning volume (veh/hr) for example for right turning movement in nearside leg 1, $V_c = t_4 + l_4 + l_3$, *Vex*, far-side volume (veh/hr) for example for right turning movement in nearside leg1, $V_{ex} = L_3 + T_4$, W_{en}, nearside legs width (meter), W_c, turning legs width (meter), and W_{ex}, far-side legs width (meter). Figure 1 exhibits the diagram of traffic flow variables in the roundabout.

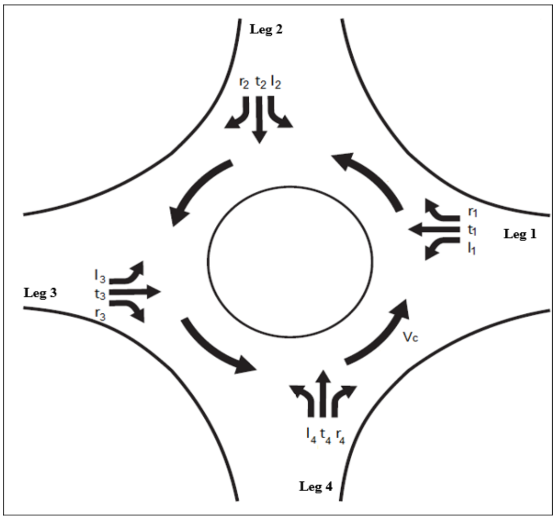

Figure 1. Traffic flow variables diagram in the roundabout

5.2. Direct Movement Model

In this section, at first, we need to set the effective variables on delay time for direct movement model as we did it for the right turning model. The effective traffic and geometric variables on direct delay time are: V_r: Right turning volume in considered nearside (veh/hr), V_{tl}: Direct movement and left turn volume in considered nearside (veh/hr), V_c = turning volume (veh/hr), for example for direct movement in nearside leg 1, $V_c = t_4 + L_4 + L_3 + r_2 + t_2 + L_2$, V_{ex}: far-side volume (veh/ hr) for example for direct movement in nearside leg 1, $V_{ex} = r_2 + L_4$, W_{ex}: nearside legs width (meter), W_c: turning legs width (meter), W_{ex}: far-side legs width (meter) and D_i: central roundabout diameter (meter).

5.3. Left turning movement model

The effective and geometric variables on left turning delay are V_{tl}: Direct movement and left turn volume in considered nearside (veh/hr), V_c= turning volume (veh/hr), for example for direct movement in nearside1, $V_c = t_4 + L_4 + L_3 + r_2 + t_2 + L_2$, V_{ex}: fareside volume (veh/ hr) for example for direct movement in nearside leg 1, $V_{ex} = r_2 + L_4$, W_{ex}: nearside legs width (meter), W_c: turning legs width (meter), W_{ex}: for-side legs width (meter) and D_i: central roundabout diameter (meter).

6. The Linear and Nonlinear Multiple Regression Modeling

In a regression model for delays in urban un-signalized roundabouts, statistical analysis methods of multiple linear and nonlinear regression models were used. The presented linear models included regression models of right turning, left turning, and direct movement as well as the dependent variable of delay in urban un-signalized roundabouts. Forward multiple regression method was used for modeling and the final version was obtained. Values of the statistical analysis including: (F), (df), and (sig.) statistics are given in Table 4. As can be seen from significant coefficient values, the relations are significant with 95 percent confidence. Fitness values of R relations are also defined with the top relation.

Table 3. Nearside and far-side legs volume and turning volume in roundabouts study

Roundabout	Right Turning Volume (Pcu/min)	Left Turning and Direct Volume (Pcu/min)	Right Turning Volume (Pcu/min)	Right Turning Volume (Pcu/min)	Delay (s)
1	43	454	512	333	18
	35	324	135	321	30
	24	686	128	470	31
	20	658	118	210	37
	40	170	346	345	21
	36	311	209	209	59
	35	658	175	415	27
	28	728	109	421	16
	42	197	356	412	11
	11	635	151	368	25
	31	563	517	494	19
	13	436	471	352	14
	36	734	523	341	38
	21	618	251	394	60
	13	555	354	421	51
	12	359	297	368	9
2	42	557	394	250	37
	21	578	176	200	45
	47	592	250	284	51
	23	612	169	307	59
	20	717	410	214	52
	29	468	193	236	64
	49	502	343	228	71
	48	508	251	237	77
	27	678	208	305	63
	37	482	233	271	67
	28	718	276	305	82
	37	457	186	207	68
	36	592	311	253	77
	42	465	389	217	58
	28	610	261	298	45
	37	561	268	298	61
3	32	173	512	333	62
	19	629	135	321	74
	47	204	128	470	41
	39	375	346	345	96
	15	252	209	209	89
	26	305	175	415	85
	40	493	356	412	77
	47	629	151	368	58
	36	594	517	494	98
	17	334	523	341	91
	20	357	251	394	48
	34	508	354	421	93

Table 4. Results of the fitness index of multiple and nonlinear regression

The Movement Model	Top Model Type	df	F	Sig.	R	Relation
Right Turning	Logarithmic	43	16.119	0.000	0.781	$D = 21.199 + 237.976 \text{ Log } w_c - 104.515 \text{ Log } v_{ex} + 72.611 \text{Log } w_{ex}$
Direct	Linear	43	32.107	0.000	0.740	$D = 299.551 + 2.661\, D_i - 42.878\, w_c$
Left Turning	Logarithmic	41	43.075	0.000	0.712	$D = -34.921 + 52.078 \text{ Log } D_i$

According to the ANOVA results of the obtained models (Table 4), the P-Value is equal to zero and hence, the significance level of these models is equal to 5% which shows the proper fit of the intended model. Results of these tests indicated validity of these models. According to the results of the fitness index of models, direct movement model is a linear relation with R = 0.74. Both turning left and turning right models are logarithmic nonlinear relations with respective coefficients of R = 0.712 and R = 0.781 . The reasons for choosing these model among other linear and nonlinear relations were the high fitness index and significant statistical levels.

7. Optimization of Models Using by Neural Network

Artificial neural networks transfer knowledge or data's hidden rule to the network structure by a process on experimental data that is why these systems are called intelligent, because on the basis of calculations on numerical data they include general rules. There are some parameters in the structure of this system that are adjustable. The reason these parameters are adjusted is that they could cope with external data and stimulations, actually this action is called system training.

Artificial neural networks have the same structures in spite of the varieties. According to Figure 2, they generally consist of input, hidden and output layers. Input layers just receive the data and acts like an independent variable, thus the number of input layers' neurons are determined by the basis of issue's nature and dependent on the number of independent variables. However, hidden layer expresses no meanings unlike input and output layers, actually it has the task of adjusting and learning the weights.

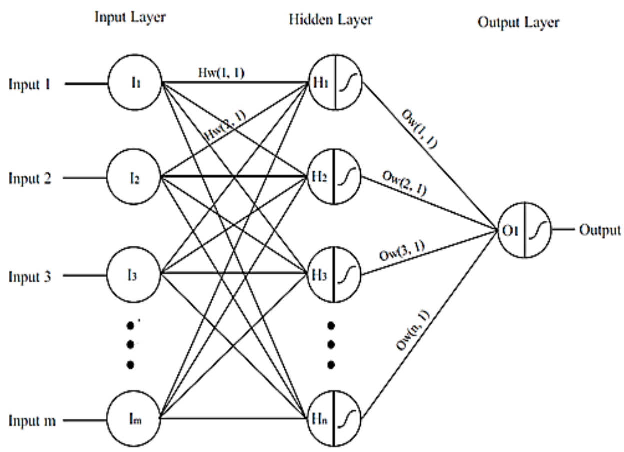

Figure 2. Artificial neural network model

Training the neural networks is actually determining optimum amounts of weights, in order to cause the system to have the least error. Thus, at first, some of the weights are selected randomly and then according to the training that includes relevant inputs and outputs and a good way of training, weights would change in the way that for amounts of input data, it would change the amounts of output data.

One of the problems that happens in neural networks training is called over fitting problem. In this situation, the error which is about training data is so low, but about new data that are submitted to the network is high. Actually, instead of learning the data generally, it memorizes the training samples. Dividing the data into two groups is one of the ways that prevents the problem. The first group is used for training and the second one is used for validating the network. The second group that includes two subgroups: validation and testing plays no rule in adjusting the parameters, but changes in error rate for both of the subgroups during the training would be considered. When error rate for the second group has increased to several repeats, training will stop and parameters will be returned to the state that the second group errors are the least.

In this modeling, due to the high number of input variables which is eight, it is decided to use the neural network for modeling. For this purpose, a network consisting eight neurons and eight hidden layers has been used. Among 44 series of data which were observed, 30 series of which were randomly selected for training the network and seven data for validation and seven data for testing the network were used. Figure 3 and 4 shows the regression models using both right turning relation and neural network with R=0.781 and R=0.914 respectively.

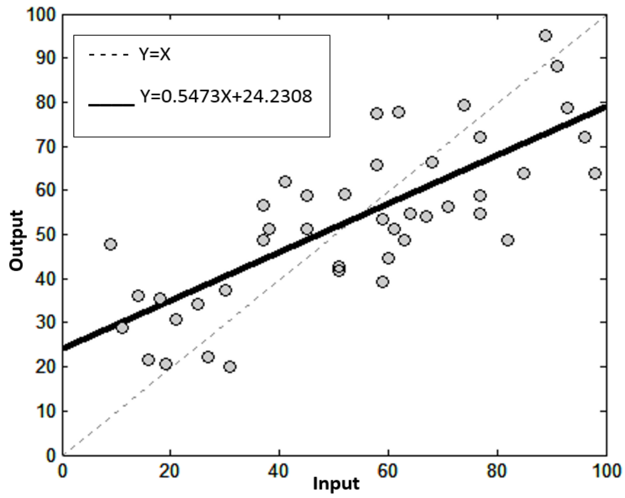

Figure 3. Regression plot using right turning relation.

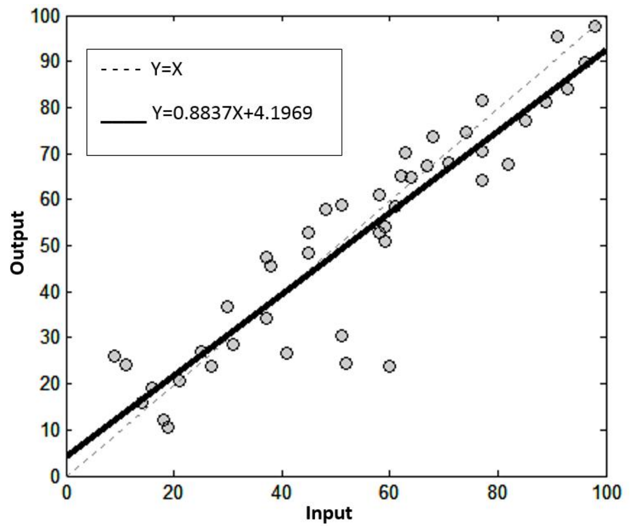

Figure 4. Regression plot using neural network

The best validation line in Figure 5 shows the best mean square error for designed network, and when the process of network training is correct that the mean square training curve is less than this amount.

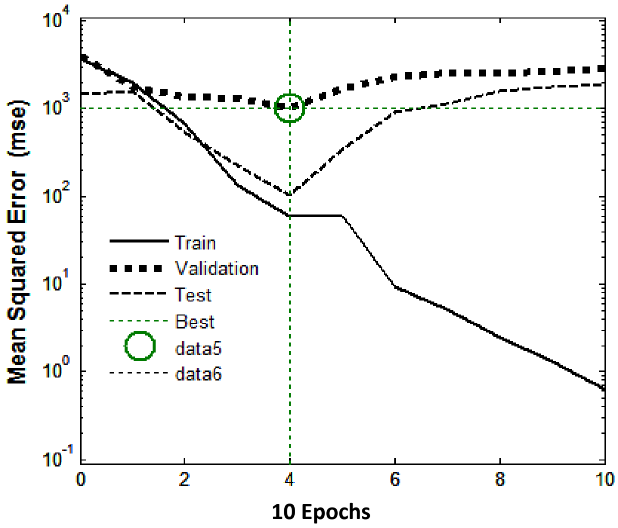

Figure 5. Neural Network Training Performance

8. Conclusions

In this study, the delay model was determined using a multivariate regression by collecting information on the volumes and the width of entrance, exit, and turning legs from three urban un-signalized roundabouts in Rasht for various movements of right turning , left turning, and direct movements. The direct movement is a linear relationship with R= 0.740. As well, both left turning and right turning movement models are logarithmic nonlinear relations with respective coefficients of R= 0.712 and R= 0.781. The reasons for selecting these models among other linear and nonlinear relationships were the high fitness index and significant statistical levels. The ability of artificial neural network has been used for modeling and prediction of delays at un-signalized roundabouts. By considering results of this modeling, it has been found that the use of artificial neural network leads to the increase of R value and it also causes a high accuracy for getting accurate outputs.

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